

GUITAR MAKING – THE ACOUSTICIAN’S TALE

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ABSTRACT

A long-standing research programme at Cardiff University has established the low- and mid-frequency mechanics and acoustics of the classical guitar. Techniques such as holographic interferometry and finite-element analysis have yielded considerable information about the modal characteristics of the instrument and their relationship with the construction and materials of the instrument. Considerable work has also been undertaken to determine the sound-radiation fields associated with these modes, establishing those modes which make the greatest contribution to the radiated energy. Studies of string dynamics (including the interaction with the player’s fingertip) show how readily the strings’ energy is coupled to the body and sound field. Our measurements and models allow a relatively small number of measured parameters to be used to predict the sounds radiated by a guitar; these sounds can be used for psychoacoustical tests to gauge those modifications to the guitar’s structure which are likely to produce perceptible differences in sound quality.

The aim of this paper is to present the key finding of this work in a form accessible for the practical maker and to present simple models which can be used by makers for effective decision making during the construction of an instrument.

1. INTRODUCTION

The most important modes of vibration of guitars are those which induce large volume changes in the surrounding air – the so-called “air-pumping modes”. The most prominent of these is the fundamental mode of the soundboard (Figure 1), which involves uni-phase motion of the lower bout. Although in the completed guitar this mode is complicated by its coupling to the air cavity and the back plate, there is much to be gained from developing simple models of this mode and investigating the factors which control its resonance frequency and also the ease with which it is excited and with which it radiates sound. The following discussions use straightforward theory (standard equations) to give a little insight into guitar design, some of which is intuitively obvious, some of which is not.

1.1 Tuning the Fundamental Mode

Calculating the modes of vibration of guitars is difficult because wood is anisotropic (i.e. it has different material properties “along” and “across” the grain), the shape of the instrument is mathematically complex, and the struts, bars and bridge are difficult to incorporate into a model. Whilst techniques such as finite element analysis allow accurate predictions to be made of mode shapes and frequencies, it is sometimes better to work with more simple models with

analytical solutions which can be used in “thought experiments”. This is the approach taken here.

The most simple approximation to the mode shown in Figure 1 is obtained by modelling the lower bout as a circular plate with a diameter roughly the width of the guitar. Further simplifications assume that the plate is made from an isotropic material and that it is unstrutted and of uniform thickness. These are not entirely unreasonable assumptions: the cross struts (“harmonic bars”) and bridge of the guitar to some extent even out the stiffness variations found “along” and “across” the grain of a flat board of spruce tone-wood, and these simplifications contrive to make the maths manageable. It turns out that the boundary conditions are important. In this first model described here it is assumed that the plate is clamped at the edges such that its displacement and slope at the boundary are both zero (this is actually a good approximation for many of the modes observed in real guitars). Solutions for the mode shapes and mode frequencies are given in many text books. The modes of this circular, isotropic plate share many of the characteristics of modes in guitars.

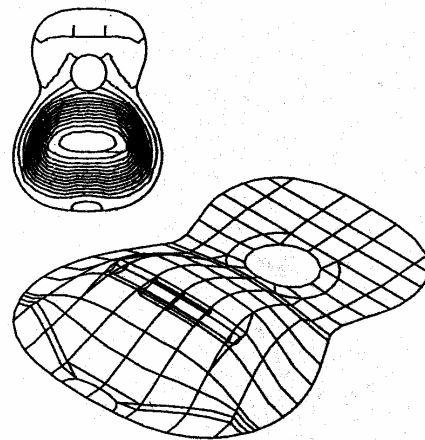


Figure 1: The fundamental mode of a guitar soundboard (finite element calculation).

The fundamental mode of a clamped circular plate is shown in Figure 2. The frequency of the mode is given by Equation 1.

$$f_{01} = 0.467 \left(\frac{h}{a^2} \right) \sqrt{\frac{E}{\rho(1-\nu^2)}}, \quad (1)$$

where h is the thickness of the plate, a its radius and ρ its volume density. E and ν are the Young’s modulus and Poisson ratio respectively (the latter is usually about 0.3 and can be ignored in these discussions). The subscript (01) refers to the mode designation: zero diametrical nodal lines and one circumferential node (in this case at the edge only).

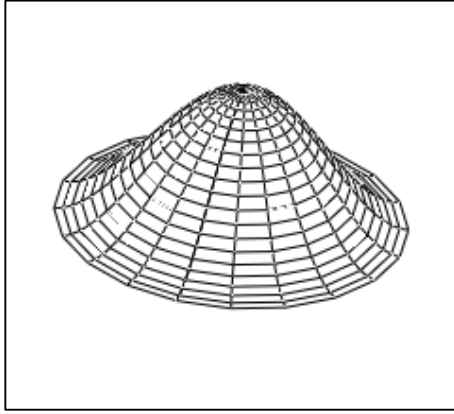


Figure 2: Fundamental mode of a plate clamped at the edge. The boundary conditions are that the displacement and slope of the vibrations are zero at the edge.

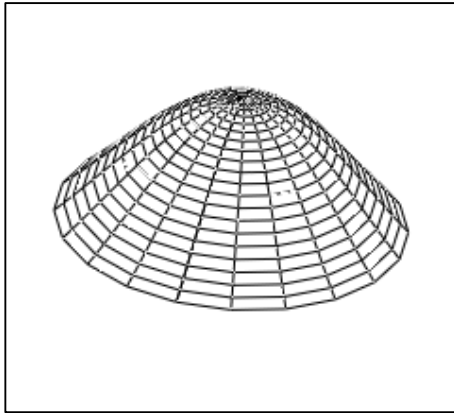


Figure 3: Fundamental mode of a plate hinged at its edge. The boundary condition is that the displacement is zero at the edge but a finite slope is allowed. (This is equivalent to the vibrations of a membrane.)

Talk of “tuning” modes implies that there is some preferred resonance frequency for this (and other) modes, and, indeed, a good deal of the scientific literature implies that mode-tuning can be used for quality control. However, it is clear that mode-tuning in isolation is not sufficient to determine the “quality” of an instrument, but it is worth noting that conventional guitar-making practice places the fundamental within a semitone or two of a “standard” position. (This is not the place to digress on mode frequencies because this discussion involves the uncoupled plate. When the plate interacts with the body cavity, two modes result both of which exhibit motion similar to Figure 1.) In the ensuing discussions, it would be easy to argue that allowing the fundamental frequency to fall could be advantageous, but significant departure from the mean can give an uncharacteristic guitar sound. There are also some arguments for keeping this mode frequency relatively high. A high fundamental ensures that the higher bending modes of the plate are well spaced throughout the playing range of the instrument and helps reduce the effects of over-coupling of string modes. Recent work at Cardiff [1] has, however, identified other parameters which we consider more important than mode frequency (such as the effective masses of modes as discussed later).

Equation 1 immediately identifies the mechanisms by which a guitar maker can control the frequency of the fundamental mode. The initial choice of materials determines E and ρ , whereas overall design (the outline shape of the guitar) determines the radius a . Once these are fixed, it leaves variations in the thickness h as the only control mechanism for tuning the mode. (In reality, thickness and strutting would be used in conjunction.)

Assuming that there is some specific mode tuning in mind, the four variables (E , ρ , a and h) offer considerable flexibility in design. For example, low-density wood or a smaller bodied instrument could be made with a thinner soundboard – and it begs the question as to whether there are “optimum” or preferred values for these quantities. Before considering these design variables, it is necessary to introduce two further equations.

1.2 Acoustic Merit of Modes

The function of the modes of the body is to act as “mediators” between the vibrating strings (which supply energy) and the surrounding air (in which sound waves are set up and energy is propagated to the listener). There are lots of subtleties in the relationships between the strings and the body and body modes and their radiation field which will not be dealt with here, but important aspects of the function of the body can be explored by examining the volume of air displaced by the body per string cycle and also the ease with which the body can be driven by the string. The latter is summarised by determining the *effective mass* of the body at the driving point (assumed to be the centre of the plate in this particular case). The effective mass is somewhat unintuitive (it can vary very substantially from the physical mass of the plate) but in physical terms it is the equivalent mass which a simple mass-spring system would have to have to exhibit the same vibrational properties as the extended mode.

The volume of air displaced by a mode vibrating transversely is given by the following integral (Equation 2).

$$V_0 = \int_{\text{area}} \psi(x, y) dA, \quad (2)$$

where $\psi(x, y)$ is the transverse displacement of the plate at a particular coordinate and the integral is performed over the whole surface. (The zero subscript is there to indicate that the integral gives the monopole contribution to the radiation only.) V_0 is basically the volume under the wire-frame figures shown in Figures 1, 2 and 3. The effective mass of the plate is also given by an integral equation.

$$M = \int_{\text{area}} \psi^2(x, y) \rho h dA. \quad (3)$$

Note that this time ψ appears as a *squared* value. This is significant, as will become evident. For the case of the circular isotropic plate clamped at its edges as shown in Figure 2 $V_0 = 0.313\pi a^2$ and $M = 0.184\pi a^2 \rho h$.

The ratio of V_0/M is a useful measure of the effectiveness of the mode to radiate energy from the string to its surroundings. For these discussions the ratio V_0/M will be called the “acoustic merit” of the mode – this is not a standard term, but it is useful to give it a name. Note that in this case the acoustic

merit is proportional to $1/\rho h$. The acoustic merit depends quite sensitively on the *shape* of the mode (i.e. $\psi(x, y)$).

Returning to Equation 1, it is clear that it is advantageous to choose values of E , ρ , h and a which *simultaneously* tune the mode and maximise the value $1/\rho h$. We now have some definite objectives with which to work.

1.3 Discussion

It is immediately obvious why “tone wood” is characterised by a high ratio of E/ρ . Spruce and cedar naturally offer some of the highest available values of this ratio. For a given size of instrument and a preferred tuning of the fundamental, a high value of E/ρ allows h to be made as small as possible thereby increasing the acoustic merit. Correct cutting of timber is essential for maintaining a maximum value of E (the fibres must be parallel to the surface of the board and the rings exactly at right angles), but growth conditions affect both E and ρ . The equations suggest that if there is a choice between material with a high Young’s modulus and high density or a low Young’s modulus and low density (E/ρ being constant), the latter would be preferable as both h and ρ could be minimised simultaneously. (This is assuming that a major criterion of guitar construction is to make an instrument which is responsive and an efficient radiator – in simple terms, and without prejudice, a “loud” instrument.) In a real guitar, the use of strutting allows the maker to maintain the stiffness in the plate (equivalent to E) whilst keeping the mass of the plate (effectively ρ) to a minimum, highlighting the acoustical advantage of using a strutted plate. Unfortunately, the relationships between plate thickness and strutting height are not so easy to investigate.

It is often suggested that large plates (large-bodied instruments) produce louder instruments, but the analysis here implies the contrary (though there must be some practical limits to how “small” the plate might be made). Note that the acoustic merit does not depend on the radius, but if a is reduced, h must also be reduced to maintain the same mode frequency. This in turn increases the acoustic merit. So why not make smaller instrument? Well, many makers do! However, note that in Equation 1 a is squared. Thus, a 10% reduction in a requires a 20% reduction in h – and the soundboard could soon get uncomfortably thin and mechanically unviable! This is particularly true of a strutted plate. Also, if the maker departs a long way from “conventional size”, for the same string length, the bridge position would move to a less active part of the soundboard. However, it is interesting to see a convincing argument against increasing the size of the instrument.

1.4 Subtleties – Mode Shape

The geometry of the plate, its boundary conditions and its elastic properties uniquely define the mode shapes. In a real guitar soundboard there is considerable choice of shape and strutting patterns and considerable variability in material properties – hence there are variations in mode shapes from one instrument to another. The positions of nodal lines relative to the bridge have a major influence on the acoustical function of the body, but even subtle changes in shapes of modes which have antinodes near the bridge, such as the fundamental, can have an impact on the workings of the instrument.

The acoustic merit involved the ratio between Equations 2 and 3, both of which involve the mode shape $\psi(x, y)$. Because $\psi(x, y)$ is squared in one equation and not in the other, the acoustic merit actually depends on $\psi(x, y)$ as well as ρ and h . This is best illustrated by a specific example.

Figures 2 and 3 show the fundamental mode of an isotropic plate under two boundary conditions: fixed (as defined previously) and “hinged”. The latter has a zero displacement at the boundary but is free to have a finite slope. This system is equivalent to the fundamental mode of a circular membrane (a drum skin).

It is interesting to determine V_0 and M for this second configuration. For the “hinged plate” these turn out to be $0.432\pi a^2$ and $0.269\pi a^2\rho h$ respectively. There is an increased volume displacement over the fixed plate – that is very evident from the figures – but the calculations show that the effective mass has also increased. Because of the squared term in the equation for M and the nature of changes in the mode shape, the effective mass rises faster than the volume displacement. For equivalent geometries, the acoustical merit of the second configuration falls by about 6%. From a cursory glance at the wire-frame pictures in Figures 2 and 3 it would be very easy to make the mistake that the latter figure was the more effective radiator.

2. A REAL CASE STUDY

Real instruments are inevitably more complex than implied in these discussions. In particular, the vibrations induced directly in the soundboard by the vibrating strings in turn couple energy to the rest of the body, which also then vibrates and radiates; the added complication is that radiation from the different parts of the guitar are not always in phase, which has considerable effect on the far-field pressure response. Coupling can be via pressure changes within the cavity (the so-called plate-Helmholtz coupling) or via structural power flow. Sound radiation is thus a combination of pressure changes induced by motion of the soundboard, the back plate and also volume flow through the sound-hole. Whilst the soundboard is undoubtedly the most important sound-radiating element, radiation from the back and air cavity can be very substantial at times (at may even dominate at some frequencies).

At Cardiff, we have set up systems to measure various “acoustical parameters”, some of which correspond to the volume displacements and effective masses discussed earlier. By way of an example, in Figure 4 we show some comparative measurements of the equivalent mode in three guitars of quite different construction. The mode shown is the most dominant of all body modes – one often referred to as the “main body resonance”. This title is somewhat of a misnomer because the mode involves significant coupling of the air cavity of the body and also involves anti-phase motion of the back plate. (We define the phase of the motion of the soundboard and back plate relative to the centre of the body. Hence “in-phase” motion implies that the soundboard and back plate both expand outwards from the cavity inducing strong volume change. “Out-of-phase” motion implies that the two plates move in the same linear direction; the net volume change is then less.)

The interferograms shown in Figure 4 show each instrument driven at an arbitrary amplitude, but a measure of how easy each mode is to drive (from the string) can be determined from the effective mass measurements quoted below. By contrast, the

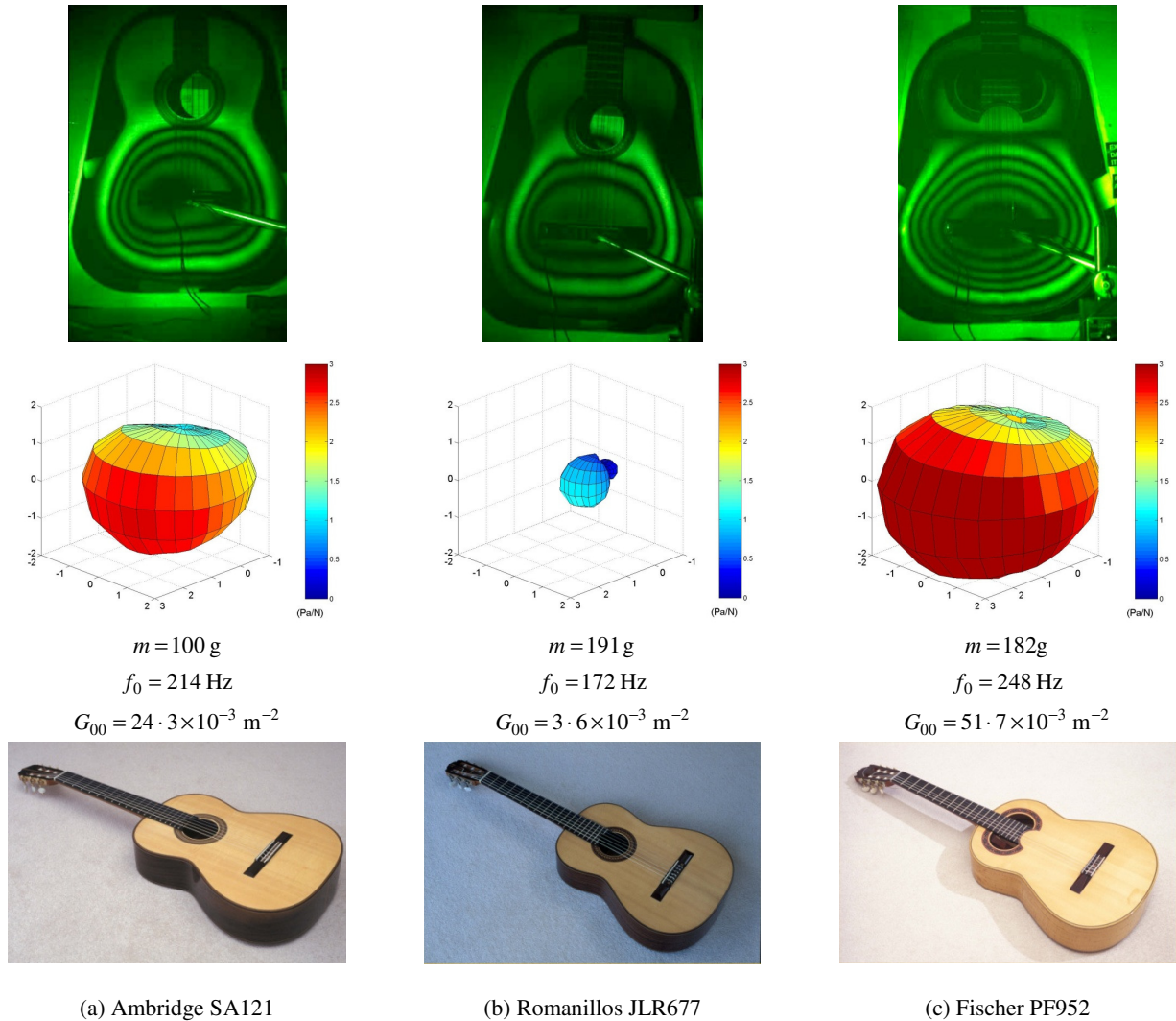


Figure 4: Comparative measurements of modes and radiation fields for three guitars of different construction (makers Simon Ambridge, José Romanillos and Paul Fischer).

sound radiation plots use the same scaling. The latter show an equal pressure surface in space. It's clear in each case that the sound radiation is largely monopole, though the techniques used also extract the higher-order-pole radiation which is responsible for the directivity, which is especially observed at progressively higher frequencies.

The acoustic merit in this case is given by the ratio of the monopole radiativity (G_{00}) to the effective mass. An interesting comparison can be made between the Ambridge and Fischer instruments. The former is a “traditional” Torres-style fan-braced instrument, whereas the latter employs a “lattice bracing” system with some clear unconventional design. The increased stiffness of the soundboard towards the periphery of the edge of the plate in this lattice-braced guitar shows the sort of “mode confinement” evident in Figure 2 compared with Figure 3. (The confinement is even more evident in higher-order modes.) The acoustic merit of this instrument is a little higher than the traditionally-braced instrument. The Romanillos instrument shows a much lower value of acoustic merit (for this mode), but this is because of over-coupling between the soundboard and back plate. The out-

of-phase radiation from the back tends to reduce the monopole contribution. Further details of these instruments and the other acoustical parameters are given by Richardson *et. al* [2].

3. ACKNOWLEDGEMENTS

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4. REFERENCES

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